An Argument against General Validity?*

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Abstract

We show that a prominent – and oft-thought to be persuasive – argument against general validity as the best account of validity for languages containing the actuality operator is flawed, the flaw arising out of inadequate attention to mood distinctions.

1 Introduction

Here we will be considering a potential problem for a particular account of validity for languages which contain the actuality operator ("@"). The languages under consideration here will predominately be propositional ones, as these are all that is needed to make the relevant points, although we will examine some examples from the perspective of quantified modal languages. In our discussion we will be concerned with *sequents*, which we take to consist of a set of formulas on the left-hand side, and a single formula on the right hand side – throughout writing the more natural $A_1, \ldots, A_n \succ B$ instead of $\{A_1, \ldots, A_n\} \succ B$.

In discussions of languages containing actuality operators a distinction is usually drawn between two kinds of validity. On the one hand we can say that a sequent $A_1, \ldots, A_n \succ B$ is valid whenever if the formulas A_i are all true at the actual world in the model, then B is also true at the actual world. Alternatively, we can say that a sequent is valid whenever if the formulas A_i are all true at a point in the model, then B is also true at that point in the model. Following the terminology used in Davies & Humberstone (1980) we will call the first of these notions of validity *real-world validity*, and the second *general validity*.[1] It is sometimes useful to think of general validity in terms of models which single out an additional world, possibly distinct from the actual world in the model, to act as the world of evaluation, thinking of a sequent as being generally valid when B is true at the world of evaluation whenever the formulas A_i are (however

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[1]Traditional discussions of the logic of 'actually' have been conducted in the framework where we consider logics as sets of theorems, rather than as involving sequents. All the morals we draw here can be regained for the formula versions of these logics by noting that whenever a sequent $A_1, \ldots, A_n \succ B$ is valid given one of the notions of validity given in the next paragraph, then the formula $(A_1 \wedge \ldots \wedge A_n) \rightarrow B$ is also valid according to the same notion of validity, and vice versa.

that world is chosen). This way of thinking about general validity is brought up in Hanson (2006) in order deal with the problems raised in Zalta (1988) concerning general validity not corresponding to a respectable notion of logical truth. We will not be concerned with the issues raised by Zalta here, although it is useful to keep this alternative conception of general validity in mind when considering our options. Instead we will be concerned with what is taken by many to be an extremely persuasive argument against general validity and in favour of real-world validity.

2 The Argument

Consider the following simple argument in favour of real-world validity as the best explication of validity, which can be extracted from (Humberstone, 2004, p.23).

(1*) The sequent "$p \succ @p$" is *a priori* truth-preserving.

(2*) The sequent "$p \succ @p$" is real-world valid, but is not generally valid.

(3*) Validity is *a priori* truth preservation.

(C) Validity is real-world validity.

We will not be questioning the correctness of the inferential moves used here. What we will question, though, is whether we should regard the first premise of the above inference as true in any sense which is relevant to bear upon questions of what the correct (or best, or philosophically preferable) account of validity is for the logic of 'actually'.

The evidence which is usually trotted out in favour of thinking that premise (1*) is true is that we can conclude (2) below from (1) *a priori.*

The glass contains water (1)

The glass actually contains water. (2)

It seems hard to object to this as an instance of *a priori* truth preservation. What I want to argue, though, is that despite appearances this example does not count as evidence for thinking that the sequent $p \succ @p$ is *a priori* truth-preserving.

3 Paying Attention to Mood

The addition of the actuality operator into the language of modal logic has a number of uses. On the purely formal side of things it allows us to 'plug a gap' in the expressive capacities of standard modal languages. But it also allows us to formalise (grammatical) mood distinctions when giving the logical form, or indeed any formal regimentation, of sentences in English. The distinction we have in mind is that between sentences like the following.

$$\text{Laura is hungry.} \tag{3}$$

$$\text{Laura might be hungry.} \tag{4}$$

Here (3) is a sentence in the indicative mood, and (4) an instance of a sentence which we will say is in the subjunctive mood. Here we are, following Wehmeier (2004), grouping together a number of different grammatical phenomena which signal that a sentence is to be evaluated non-indicatively, the presence of which are often signalled by the verb-mood in a sentence in English being either subjunctive or conditional.

Consider the language of first-order **S5**. On the standard way of thinking (called the 'Scope Theory of Mood' in Wehmeier (2004)) a sentence like (3) would be given the same logical form as it is given in non-modal predicate logic, namely:

$$\textsc{Hungry}(Laura). \tag{5}$$

In general, on this view, we will deal with the indicative/subjunctive mood distinction using the scope of the modal operators. So for example, the sentence "Laura is Hungry, and Michael might be too" would, on this approach be rendered as the following formula.

$$\textsc{Hungry}(Laura) \wedge \Diamond\textsc{Hungry}(Michael). \tag{6}$$

As is shown in §§2 and 3 of Wehmeier (2004) this attempt to handle mood distinctions in English will not do, as there are sentences like the following for which it will deliver the wrong truth conditions.

$$\text{Everything which is red could be shiny.} \tag{7}$$

Indeed, one the primary motivations for the introduction of the 'actuality' operator given in Crossley & Humberstone (1977) is to enrich the language of modal predicate logic in order to be able to give the correct truth conditions for such sentences.[2] In order to give the correct truth conditions for such sentences a device like the 'actuality' operator is needed.

Using the 'actuality' operator, we are able to better handle the mood distinctions in English, giving the logical form for sentences like (3) and (4) as follows.

[2] More discussion on why such a device is needed in connection with the phenomena there called *any-scattering* can be found in (Humberstone, 1982, p.99). The arguments of this section are inspired by comments made in Wehmeier (2004). Wehmeier himself favours a different way of registering the indicative/non-indicative distinction, comments comparing the two approaches can be found in (Wehmeier, 2005, p.200) as well as the discussion in (Humberstone, 2004, p.46-49).

$$@\textsc{Hungry}(Laura). \tag{8}$$

$$\Diamond\textsc{Hungry}(Laura). \tag{9}$$

Here the actuality operator is used in (8) to signal that the predicate Hungry is to be evaluated indicatively. Like sentence (3), the sentence (1) above is a sentence in the indicative mood, and so in giving a regimentation of it into the language of first-order modal logic we should be using the following formula.

$$@(\textsc{Contains}(glass, water)). \tag{10}$$

The reason sentences like (1) and (2) are regarded as evidence for the truth of (1*) above is that the inferential move involved in moving from one to the other has nothing to do with the subject matter of either sentence. This is true, but in order for the inference from (1) to (2) to be formalisable as the sequent $p \succ @p$ we have to make some assumptions concerning the interpretation of the propositional variables (and more generally, atomic formulas) in the logic of 'actually'.

Standardly, in the logic of 'actually' a propositional variable p_i is true at a world w in a model if and only if w is in the truth set of p_i – the evaluation of other atomic formulas following similarly. That is, the evaluation of atomic formulas depends on the world of evaluation, while the evaluation of indicative sentences do not – all they depend upon is what is going on at the actual world. As it is put in (Humberstone, 2004, p.48) in the course of showing how a language under consideration there agrees with a language containing the 'actuality' operator "the presumption is of subjunctivity unless there is explicit 'indicativization"'. Given this, it looks as if rendering the inference from (1) to (2) as being of the form $p \succ @p$ is somewhat disingenuous.

There are a few responses which could be made at this stage. Firstly one might argue that the atomic formulas of our modal language should be regarded as being 'moodless' or 'neutral' between subjunctive and indicative readings. In this case, though, it is no longer clear what exactly one is accepting when one counts $p \succ @p$ as valid, or what would count as evidence for its validity, and accordingly we will put this option to one side. Equally, we could not simply have all of our atomic expressions be regarded as being in the indicative. Consider the following sentence.

It is possible that Laura is hungry. (11)

If we were to think of all of our atomic expressions as being indicative then this, and not (4) above, would be the reading of $\Diamond\textsc{Hungry}(Laura)$. As is pointed out in (Wehmeier, 2004, p.608) such a sentence suggests an epistemic reading (i.e. that I cannot rule out that Laura is hungry) rather than a metaphysical one.

Alternatively one could try and avoid the issue in question, claiming that these problems do not occur if we are evaluating sentences for their truth at the actual world. In this case all that the truth of a propositional variable depends on is what is going on at the actual world. But this move is just to beg the question against the defender of general validity. What this proposal amounts to is the suggestion that in order for a formula to be true in a model it must be true at the actual world. I take it that both sides in the dispute here agree that a sequent is valid when for all models for that language if the premises are true in the model, then the conclusion is also. Indeed one of the points on which Zalta (1988) criticises general validity is that it violates this constraint, this challenge being met in Hanson (2006). Given this, then, the way out being suggested here just amounts to assuming that validity for sequents just is real-world validity! Dialectically this will not do: after all, this was meant to be an argument in favour of thinking of validity as real-world validity – showing that general validity cannot account for inferences which we think are intuitively valid (i.e. *a priori* truth-preserving).

So it seems that, if we are to be faithful to our semantic story in marking those formulas which are to be evaluated indicatively by prefixing them with @, then (1), being in the indicative mood, should be formalised as "$@p$". Given this, how are we to justify the inference in question? Two ways of formalising this inference suggest themselves. On the one hand we can use the actuality operator solely as a marker of the indicative mood, prefixing it to expressions which are to be evaluated indicatively, and treating occurrences of the adverb "actually" as merely providing emphasis. Following this manner of converting natural language inferences to sequents, the inference from (1) to (2) becomes the sequent $@p \succ @p$. Alternatively, we could take an approach where in addition to using the actuality operator to track mood distinctions, we are also treating it as the formal analogue of the adverb "actually" – in which case the inference from (1) to (2) becomes the sequent $@p \succ @@p$.[3] Both of these sequents are generally valid (and so *a fortiori* also real-world valid). Thus, if we properly take mood into account we can see that general validity can deal with our conception of validity as *a priori* truth-preservation just as well as real-world validity can. Moreover, if this account is correct, then it appears that it is real-world validity which does not properly accord with our intuitions concerning *a priori* truth-preservation, as it regards the sequent $p \succ @p$ as valid – which is roughly equivalent to the inference from the dubiously grammatical "Were it the case that the glass contained water" to "The glass actually contains water", which is clearly not *a priori* truth-preserving.[4]

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[3]Note that this solution exploits the two motivations for introducing the actuality operator distinguished above: both as an explicit formal analogue of the word "actually", and also as a mood indicator.

[4]The more grammatically edifying version of the above inference would be something more like that from "Under certain circumstances, the glass would have contained water" to "The glass actually contains water". It does seem as if the first sentence would be better formalised as $\Diamond p$ rather than p, though.

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