Expressive Power, Mood, and Actuality*

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Abstract

In Wehmeier (2004) we are presented with the subjunctive modal language, a way of dealing with the expressive inadequacy of modal logic by marking atomic predicates as being either in the subjunctive or indicative mood. Wehmeier claims that this language is expressively equivalent to the standard actuality language, and that despite this the marked-unmarked dichotomies are not the same in the two languages. In this paper we will attend to Wehmeier's argument that this is the case, and show that this conclusion rests on what might be considered an uncharitable stipulation concerning what it is for a formula in the actuality language to be true in a model.

1 Introduction

The expressive limitations of the standard language of modal logic are well known. The traditional way of fixing this problem of expressive inadequacy is to enrich our language with the addition of an actuality operator A as is done in, for example, Crossley & Humberstone (1977) and Davies & Humberstone (1980), but this is not the only way of solving this problem. Rather than singling out those sentences which are to be evaluated indicatively, as is done by the actuality operator, and make the default mood of formulas subjunctive, we could instead single out those which are to be evaluated subjunctively, and make the default mood of formulas indicative. This is the approach taken in Humberstone (1982) where the prospects of adding a subjunctive operator S to the language of standard modal logic are investigated – resulting in the language we will refer to as L_{SS} when we need to discuss it later.[1] A similar approach to this is taken in Wehmeier (2004) where, rather than introducing a new operator to the language of standard modal logic, it is proposed that we instead add another sort of atomic predicates to our language, marked with a superscript s

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[1] Wehmeier raises some issues relating to favouring L_{SS} over his Subjunctive Modal Logic in (Wehmeier, 2005, p.201), which are discussed in (Humberstone, 2004, p.46f). The language which we have been calling L_{SS} is called L_C in Humberstone (1982) due to connections between this kind of language and some considerations of H.-N. Castañeda's which are discussed in §2 of Humberstone (1982).

– the old sort of atomic expressions to be evaluated indicatively, and the new ones subjunctively.

It appears as if the only difference between these two languages is which mood is marked and which is left unmarked – the actuality language marking the indicative mood, and the subjunctive language the subjunctive mood. Wehmeier claims, though, that while his language is expressively equivalent to the actuality language, it nonetheless differs in its marked/unmarked dichotomies. What we will argue here is that this difference in the marked-unmarked dichotomies in the two languages is due to an artefact of Wehmeier's treatment of the actuality language. To show this first, in §2 we set the scene for the ensuing discussion – giving the details of the two languages and the expressive equivalence result from Wehmeier (2004). In §3 we explicate Wehmeier's comments concerning the difference in marked-unmarked distinctions between the two languages, and argue that Wehmeier is too hasty in putting aside a potential objection, before finally in §4 looking at where this leaves the relationship between the actuality language and Wehmeier's subjunctive language.

2 The Logic of Actually and Wehmeier's Subjunctive Modal Logic

In the interests of simplicity we consider only the propositional versions of these two languages, noting that our argument will extend to the first-order case with very little effort. The first language L_S will be constructed out of denumerably many (indicative) propositional variables $p_1, \ldots, p_n, \ldots$, denumerably many (subjunctive) propositional variables $p_1^s, \ldots, p_n^s, \ldots$ – each of which corresponds to the appropriate indicative propositional variable – using the boolean connectives $\{\wedge, \neg\}$ (and, not) and the modal operator $\Diamond$ (possibility). The second language L_A will be constructed out of denumerably many propositional variables $p_1, \ldots, p_n, \ldots$ using the boolean connectives $\{\wedge, \neg\}$ (and, not) and the modal operators $\Diamond$ (possibility), and A (actually).[2] Say that a formula in L_S is *subjunctively closed* iff every occurrence of a subjunctive propositional variable p_i^s lies within the scope of a modal operator. So, for example,the formula $\Diamond p^s$ will be subjunctively closed, while the formula $\Diamond p^s \wedge q^s$ will not be.

Our semantic structures will be simplified Kripke models for **S5** ($\langle W, @, V\rangle$) with a distinguished point $@ \in W$ corresponding to the actual world. Truth of a formula φ at a point x in a model $\mathcal{M} = \langle W, @, V\rangle$ according to L_S ("$\mathcal{M} \Vdash_x \varphi$")

[2] We address any worries one might have about the potential redundancy of the actuality operator in propositional modal logics in note 6 below.

will be defined inductively as follows.

$$
\begin{aligned}
\mathcal{M} \Vdash_x p_i &\iff @ \in V(p_i).\\
\mathcal{M} \Vdash_x p_i^s &\iff x \in V(p_i).\\
\mathcal{M} \Vdash_x \varphi \wedge \psi &\iff \mathcal{M} \Vdash_x \varphi \text{ and } \mathcal{M} \Vdash_x \psi.\\
\mathcal{M} \Vdash_x \neg\varphi &\iff \mathcal{M} \nVdash_x \varphi.\\
\mathcal{M} \Vdash_x \Diamond\varphi &\iff \exists y \in W \text{ s.t. } \mathcal{M} \Vdash_y \varphi.
\end{aligned}
$$

Similarly, we will define truth of a formula φ at a point x in a model $\mathcal{M} = \langle W, @, V\rangle$ according to L_A ("$\mathcal{M} \models_x \varphi$") inductively as follows.

$$
\begin{aligned}
\mathcal{M} \models_x p_i &\iff x \in V(p_i).\\
\mathcal{M} \models_x \varphi \wedge \psi &\iff \mathcal{M} \models_x \varphi \text{ and } \mathcal{M} \models_x \psi.\\
\mathcal{M} \models_x \neg\varphi &\iff \mathcal{M} \not\models_x \varphi.\\
\mathcal{M} \models_x \Diamond\varphi &\iff \exists y \in W \text{ s.t. } \mathcal{M} \models_y \varphi.\\
\mathcal{M} \models_x \mathsf{A}\varphi &\iff \mathcal{M} \models_@ \varphi.
\end{aligned}
$$

One important fact about subjunctively closed formulas which we will need to make use of below is that they are invariant in the sense recorded by the following Lemma.

Lemma 1. *Suppose that φ is a subjunctively closed formula from L_S. Then for all models $\mathcal{M} = \langle W, @, V\rangle$ and all points $x, y \in W$ we have the following.*

$$\mathcal{M} \Vdash_x \varphi \textit{ if and only if } \mathcal{M} \Vdash_y \varphi.$$

Intuitively speaking, two languages are expressively equivalent when they are able to discriminate between the same models. In the particularly strong sense which will be at issue here, this will mean that languages L_1 and L_2 are expressively equivalent when for every formula $\varphi \in L_1$ there is a formula $\psi \in L_2$ such that φ and ψ are true in the same models. Writing $\mathcal{M} \models_{L_i} \varphi$ to mean (where $\varphi \in L_i$) that $\varphi \in L_i$ is true in the L_i sense in the model $\mathcal{M}$, this leaves us with the following definition.

Definition 1. Two languages L_1 and L_2 are expressively equivalent iff we have the following conditions satisfied

$$
\begin{aligned}
&\forall\varphi \in L_1 \exists\psi \in L_2 (\forall\mathcal{M} : \mathcal{M} \models_{L_1} \varphi \iff \mathcal{M} \models_{L_2} \psi).\\
&\forall\psi \in L_2 \exists\varphi \in L_1 (\forall\mathcal{M} : \mathcal{M} \models_{L_1} \varphi \iff \mathcal{M} \models_{L_2} \psi).
\end{aligned}
$$

Applying this notion to pairs of languages requires us to be clear on what it is for a formula to be true in a model – otherwise the notion $\mathcal{M} \models_{L_i} \varphi$ will be undefined. In the modal case we have two obvious candidates for what it is for a formula φ to be true in a model $\mathcal{M} = \langle W, @, V\rangle$. On the one hand we could say that a formula is true in a model whenever it is true at the actual world in that model – call this *real-world truth*. On the other hand we could say that a formula is true in a model whenever it is true at all worlds in that model – call this *general truth*. Given a notion of truth at a point in a model $\mathcal{M} \models_x \varphi$, we will denote φ being real-world true as $\mathcal{M} \models^r \varphi$, and it being generally true as $\mathcal{M} \models^g \varphi$. This terminology is an adaptation of the distinction between real-world validity and general validity from Davies & Humberstone (1980). Using this terminology a formula φ will be *real-world valid* when it is real-world true in all models, and *generally valid* when it is generally true in all models. We will not have much to say about general truth here, although we will have quite a bit to say later about a further way of thinking about truth in a model which also yields general validity in the same way general truth does.

Wehmeier is very clear that truth in a model for formulas in L_A is to be understood as real-world truth – referring to it as the "received sense" of truth in a model for L_A. Wehmeier also says that, for subjunctively closed formulas at least, real-world truth and general truth coincide (this following in the obvious way from Lemma 1). Accordingly we will, for the moment, regard truth in a model as real-world truth for formulas in L_S which are subjunctively closed. So, according to Wehmeier we have truth in a model for formulas of L_A being defined as $\models^r$, and truth in a model for subjunctively closed formulas of L_S being defined as $\Vdash^r$. Given this understanding of what it is for a formula to be true in a model we can now investigate whether these two languages are expressively equivalent.[3]

In order to show that these two languages are expressively equivalent we need to be able, given a formula in one language, to construct a formula in the other such that these formulas are true in the same models. One obvious way of doing this is to use a translation τ, constructed so that given a formula φ in one language $\tau(\varphi)$ is a formula in the other language which is true in the same models as φ. To this end, Wehmeier proposes two translations – rendered here as τ_1 and τ_2.[4] The translation τ_1 maps formulas of L_A to subjunctively closed formulas of L_S. One odd feature of τ_1 is that it is given in terms of an auxiliary translation τ_S, which will do some important work in what is to come.

$$\begin{aligned} \tau_1(p_i) &= p_i & \tau_S(p_i) &= p_i^s. \\ \tau_1(\varphi \wedge \psi) &= \tau_1(\varphi) \wedge \tau_1(\psi) & \tau_S(\varphi \wedge \psi) &= \tau_S(\varphi) \wedge \tau_S(\psi) \\ \tau_1(\neg\varphi) &= \neg\tau_1(\varphi) & \tau_S(\neg\varphi) &= \neg\tau_S(\varphi) \\ \tau_1(\Diamond\varphi) &= \Diamond\tau_S(\varphi) & \tau_S(\Diamond\varphi) &= \Diamond\tau_S(\varphi) \\ \tau_1(\mathsf{A}\varphi) &= \tau_1(\varphi) & \tau_S(\mathsf{A}\varphi) &= \tau_1(\varphi) \end{aligned}$$

Given this translation, then, we have the following result.

Lemma 2. *For all models $\mathcal{M} = \langle W, @, V \rangle$ and all formulas $\varphi \in L_A$ we have the following for all points $y \in W$.*

$$\mathcal{M} \models_@ \varphi \iff \mathcal{M} \Vdash_@ \tau_1(\varphi) \text{ and } \mathcal{M} \models_y \varphi \iff \mathcal{M} \Vdash_y \tau_S(\varphi).$$

[3] The following two results are stated without proof in (Wehmeier, 2004, p.619).

[4] These are the obvious recursive definitions of the translations suggested by Wehmeier's remarks on p.619 of Wehmeier (2004).

Proof. By induction on the complexity of φ. For the basis case it is easy to see that for all $y \in W$ we have $\mathcal{M} \models_@ p_i \iff \mathcal{M} \Vdash_@ p_i$, and also that $\mathcal{M} \models_y p_i \iff \mathcal{M} \Vdash_y p_i^s$.

For convenience we label the two parts of the induction hypothesis, for all ψ of lower complexity than that of the formula currently under consideration, for any $y \in W$.
Induction Hypothesis-i: $\mathcal{M} \models_@ \psi \iff \mathcal{M} \Vdash_@ \tau_1(\psi)$.
Induction Hypothesis-s: $\mathcal{M} \models_y \psi \iff \mathcal{M} \Vdash_y \tau_S(\psi)$.

Only the inductive cases for $\Diamond$ and A need to be considered, the case for the boolean connectives being routine.

Suppose now that $\varphi = \Diamond\psi$.
Part 1: Suppose that $\mathcal{M} \models_@ \Diamond\psi$. Then there is a $z \in W$ such that $\mathcal{M} \models_z \psi$. By Induction Hypothesis-s it follows that $\mathcal{M} \Vdash_z \tau_S(\psi)$, and so in particular that $\mathcal{M} \Vdash_@ \Diamond\tau_S(\psi)$, which is just $\tau_1(\Diamond\psi)$ as desired. Suppose now that $\mathcal{M} \not\models_@ \Diamond\psi$. Then for all $z \in W$ we have $\mathcal{M} \not\models_z \psi$. So by Induction Hypothesis-s it follows that $\mathcal{M} \nVdash_z \tau_S(\psi)$, and so it follows that $\mathcal{M} \nVdash_@ \Diamond\tau_S(\psi)$ – which is $\tau_1(\Diamond\psi)$ as desired.
Part 2: Suppose that $\mathcal{M} \models_y \Diamond\psi$ Then there is a $z \in W$ such that $\mathcal{M} \models_z \psi$. By Induction Hypothesis-s it follows that $\mathcal{M} \Vdash_z \tau_S(\psi)$, and hence that $\mathcal{M} \Vdash_y \Diamond\tau_S(\psi)$ as desired. Suppose now that $\mathcal{M} \not\models_y \Diamond\psi$. Then for all $z \in W$ we have that $\mathcal{M} \not\models_z \psi$. So by Induction Hypothesis-s it follows that $\mathcal{M} \nVdash_z \tau_S(\psi)$, and so it follows that $\mathcal{M} \nVdash_y \Diamond\tau_S(\psi)$ as desired.

Suppose now that $\varphi = \mathsf{A}\psi$.
Part 1: Suppose that $\mathcal{M} \models_@ \mathsf{A}\psi$. Then it follows that $\mathcal{M} \models_@ \psi$. Hence by Induction Hypothesis-i it follows that $\mathcal{M} \Vdash_@ \tau_1(\psi)$ as desired. Suppose now that $\mathcal{M} \not\models_@ \mathsf{A}\psi$. Then it follows that $\mathcal{M} \not\models_@ \psi$, and hence by Induction Hypothesis-i it follows that $\mathcal{M} \nVdash_@ \tau_1(\psi)$ as desired.
Part 2: Suppose that $\mathcal{M} \models_y \mathsf{A}\psi$. Then it follows that $\mathcal{M} \models_@ \psi$. Hence by Induction Hypothesis-i we have that $\mathcal{M} \Vdash_@ \tau_1(\psi)$, and as this formula is subjunctively closed by Lemma 1 it follows that $\mathcal{M} \Vdash_y \tau_1(\psi)$, which is $\tau_S(\mathsf{A}\psi)$. Suppose now that $\mathcal{M} \not\models_y \mathsf{A}\psi$. Then it follows that $\mathcal{M} \not\models_@ \psi$, and hence by Induction Hypothesis-i we have that $\mathcal{M} \nVdash_@ \tau_1(\psi)$. As this formula is subjunctively closed it follows by Lemma 1 that $\mathcal{M} \nVdash_y \tau_1(\psi)$ as desired. □

Corollary 1. *For all models $\mathcal{M} = \langle W, @, V\rangle$ and all $\varphi \in L_A$ we have the following.*

$$\mathcal{M} \models^r \varphi \text{ if and only if } \mathcal{M} \Vdash^r \tau_1(\varphi).$$

For the next Lemma we will need the following translation τ_2 which maps formulas of L_S to formulas of L_A.

$$\begin{aligned}
\tau_2(p_i) &= \mathsf{A}p_i \\
\tau_2(p_i^s) &= p_i \\
\tau_2(\varphi \wedge \psi) &= \tau_2(\varphi) \wedge \tau_2(\psi) \\
\tau_2(\neg\varphi) &= \neg\tau_2(\varphi) \\
\tau_2(\Diamond\varphi) &= \Diamond\tau_2(\varphi).
\end{aligned}$$

Lemma 3. *For all models $\mathcal{M} = \langle W, @, V\rangle$ and all formulas $\varphi \in L_S$ we have the following, for all $x \in W$.*

$$\mathcal{M} \Vdash_x \varphi \text{ if and only if } \mathcal{M} \models_x \tau_2(\psi).$$

Proof. By induction upon the complexity of φ. For the basis case note that we have $\mathcal{M} \Vdash_x p_i^s \iff \mathcal{M} \models_x p_i$, and also $\mathcal{M} \Vdash_x p_i \iff \mathcal{M} \models_@ p_i \iff \mathcal{M} \models_x \mathsf{A}p_i$.

For our induction hypothesis suppose that for all formulas ψ of complexity less that φ we have that $\mathcal{M} \Vdash_x \psi \iff \mathcal{M} \models_x \tau_2(\psi)$.

Suppose, then, that $\varphi = \Diamond\psi$, and that $\mathcal{M} \Vdash_x \Diamond\psi$. Then we know that there is a $y \in W$ such that $\mathcal{M} \Vdash_y \psi$. By the inductive hypothesis it follows that $\mathcal{M} \models_y \tau_2(\psi)$, and hence that $\mathcal{M} \models_x \Diamond\tau_2(\psi)$ as desired. Suppose now that $\mathcal{M} \models_x \Diamond\tau_2(\psi)$. It follows then that $\mathcal{M} \models_z \tau_2(\psi)$ for some $z \in W$. By the induction hypothesis it follows that $\mathcal{M} \Vdash_z \psi$, and thus that $\mathcal{M} \Vdash_x \Diamond\psi$ as desired. $\square$

Corollary 2. *For all models $\mathcal{M} = \langle W, @, V\rangle$ and all $\varphi \in L_S$ we have the following.*

$$\mathcal{M} \Vdash^r \varphi \text{ if and only if } \mathcal{M} \models^r \tau_2(\varphi).$$

Proposition 1 (Wehmeier (2004))**.** *Let truth in a model for formulas in L_A and L_S be real world truth. Then L_A and L_S are expressively equivalent.*

Proof. We need to show that, for formulas $\varphi \in L_A$, and $\psi \in L_S$ we have the following.

$$\forall\varphi\exists\psi(\forall\mathcal{M} : \mathcal{M} \models^r \varphi \iff \mathcal{M} \Vdash^r \psi). \tag{1}$$

$$\forall\psi\exists\varphi(\forall\mathcal{M} : \mathcal{M} \models^r \varphi \iff \mathcal{M} \Vdash^r \psi). \tag{2}$$

Letting our formula ψ be $\tau_1(\varphi)$ in (1), and letting our φ be $\tau_2(\psi)$ in (2) the result follows directly from Corollaries 1 and 2. $\square$

It is worth noting that the above expressive equivalence result also holds when we restrict the formulas $\psi \in L_S$ to those which are subjunctively closed – as τ_1 translates formulas of L_A into formulas of L_S which are subjunctively closed (as noted above), and Corollary 2 holds for all formulas (and so in particular for all subjunctively closed formulas).

3 Truth in a Model

One might argue that we should find the expressive equivalence of L_A and L_S completely unsurprising, as the only difference between these two languages is that in L_A we have marked indicatives and unmarked subjunctives, and in L_S we have unmarked indicatives and marked subjunctives. Wehmeier's view on this issue is made quite clear in the following quotation. Here Wehmeier is using AML to denote L_A and SML to denote L_S. We have changed his example, which involved the translation of the quantifiers, to a propositional one – the end result is the same.

> "It is important to realize, however, that [L_S and L_A] are not simply notational variants of each other – one might think that the sole difference consists in that AML marks indicative predicates and has unmarked subjunctive predicates, whereas SML has unmarked indicatives and marked subjunctives. But this is not quite so. For instance, the translation procedure maps the two formulae $[p]$ and $[\mathsf{A}p]$ of AML to the one SML formula $[p]$; and so the marked-unmarked dichotomies are not the same in the two languages." (Wehmeier, 2004, p.619)

One might object that we could translate these two sentences as p^s and p respectively, but Wehmeier points out in a footnote that a translation scheme for which $\tau(p) = p^s$ and $\tau(\mathsf{A}p) = p$ would only work "if the subjunctive world parameter is required to be set to the actual world"(Wehmeier, 2004, p.628, fn.16).

To make this point clear it will be convenient, adapting a suggestion in (Hanson, 2006, p.442), to consider a new type of model. A *d-model* is a structure $\langle W, w^*, @, V\rangle$ exactly like the models we have been considering expanded by the addition of another possibly, but not necessarily, distinct distinguished world w^* – the *designated point* in the model. Truth at a point in a d-model for both L_A and L_S works exactly like truth at a point in a model for models of the form $\langle W, @, V\rangle$ – the only place where the new distinguished world will be used is in determining what it takes for a formula to be true in a model. The addition of a new distinguished point opens up another useful notion of what it is for a formula to be true in a model in addition to real-world and general truth mentioned above. Say that a formula φ is true in a d-model $\langle W, w^*, @, V\rangle$ whenever it is true at w^*, that is whenever it is true at the designated point. In order to talk about this notion of truth in a model in contrast to real-world and general truth we will say that whenever a formula is true at the designated point in a model it is *d-true* in that model.[5] As with general and real-world truth we will denote d-truth with a superscript d, so we will understand $\mathcal{M} \models^d \varphi$ and $\mathcal{M} \Vdash^d \psi$ to mean that $\varphi \in L_A$ (resp. $\psi \in L_S$) is d-true in the d-model $\mathcal{M}$. Like general truth, this notion of truth in a model corresponds to general validity, in that a formula is generally true in a model $\langle W, @, V\rangle$ when it is d-true in all d-models based on that model (i.e., all d-models $\langle W, w^*, @, V\rangle$ with $w^* \in W$). Thus general validity, construed as general truth in all d-models, coincides with

[5] Where 'model' here should be read as d-model.

general validity as defined above in terms of models simpliciter (as observed in (Hanson, 2006, p.443)). Henceforth we will talk in terms of d-models, but continue to refer to truth at the designated point as d-truth.

Let us now move to thinking in terms of d-models, and identify, for the moment, the default subjunctive world parameter with the designated point w^* – thinking of truth in a model for formulas in L_S as d-truth, and truth in a model for formulas in L_A as real-world truth as before. Then Wehmeier's point, is that in order for a translation τ for which $\tau(p) = p^s$ and $\tau(\mathsf{A}p) = p$ to show that L_A and L_S are expressively equivalent we would be required to make the default designated world (our default subjunctive world parameter) be the actual world – otherwise we could have $\mathcal{M} \models_{@} p$ (and hence $\mathcal{M} \models^r p$), and $\mathcal{M} \not\Vdash_{w^*} \tau(p)$ (and hence $\mathcal{M} \not\Vdash^d \tau(p)$), as we can just have $V(p) = \{@\}$ and $w^* \neq @$. Thus we see a failure of the reformulated version of (1) for $\varphi = p$ and $\psi = \tau(p)$, and so Wehmeier concludes that we cannot use such a translation to show that L_A and L_S are expressively equivalent.

This is true, as far as it goes, but this only occurs because we are thinking of truth in a model for formulas of L_A in terms of real-world truth.[6] That is to say, the reason why Wehmeier claims that a translation scheme which works like τ above will only work if we require that the default subjunctive world parameter to be the actual world is that Wehmeier is assuming that truth in a model for L_A is truth at the actual world. However, if we have no good reason for thinking that the default subjunctive parameter should be the actual world when evaluating formulas in L_S, we equally well have no good reason for thinking that the default world of evaluation for formulas of L_A (i.e. our default subjunctive world) should be the actual world. As Wehmeier notes:

> "while it may so happen that the actual world becomes the salient possible world, it would be counterintuitive to require that, in the absence of an otherwise specified world, the actual world serves by default as the salient possible world."(Wehmeier, 2004, p.628, fn.16).

Thus if this argument is good enough for us to conclude that the default world of evaluation for formulas of L_S should not be the actual world, then it is equally good enough for us to conclude the same about formulas of L_A.[7]

[6] A related phenomenon occurs when we consider whether the actuality operator is eliminable in propositional modal logic, as is argued in Hazen (1978). Hazen is there concerned with pointing out the contrast between propositional and predicate modal logics with the actuality operator, arguing that the actuality operator is eliminable in the propositional case, but not in the predicate case. The proof of the eliminability of the actuality operator given by Hazen relies upon us thinking of validity as real-world validity in order for the formula $\mathsf{A}p \leftrightarrow p$ to be a theorem – and thus, truth in a model for formulas in L_A as real-world truth. If we take the eliminability of A from L_A to imply the expressive equivalence between L_A and the A-free modal language L_M, then the same concerns as above show that this will only go through if we think of truth in a model for formulas in L_M as real-world truth. If we instead think of truth in a model for formulas in L_M as d-truth then we can show that these two languages are not expressively equivalent – let $W = \{@, w\}$, and let $V = \{\langle p, @\rangle\}$ then the models $\langle W, w, @, V\rangle$ and $\langle W, @, @, V\rangle$ have the same L_A-formulas (real-world) true in them, but different formulas in L_M d-true in them.

[7] Hanson suggest a similar line of argument on p.443 of Hanson (2006). It is worth noting that the ambiguities in logical form which L_A predicts (discussed at the end of section 4 of

If we think of truth in a model for formulas of the actually language as truth at the designated point in that model then it looks like Wehmeier's comments regarding the marked-unmarked dichotomies in the two languages no longer apply – as we no longer have distinct formulas of L_A being mapped to the same formula in L_S. Moreover the two languages are expressively equivalent in this sense.

Proposition 2. *Let truth in a d-model for formulas of L_A and L_S be d-truth. Then L_A and L_S are expressively equivalent.*

Proof. We need to show that, for formulas $\varphi \in L_A$, and $\psi \in L_S$ we have the following.

$$\forall\varphi\exists\psi(\forall\mathcal{M} : \mathcal{M} \models^d \varphi \iff \mathcal{M} \Vdash^d \psi). \tag{3}$$
$$\forall\psi\exists\varphi(\forall\mathcal{M} : \mathcal{M} \models^d \varphi \iff \mathcal{M} \Vdash^d \psi). \tag{4}$$

Letting our formula ψ be $\tau_S(\varphi)$ in (3), and letting our φ be $\tau_2(\psi)$ in (4) the result follows directly from Lemma 2 and Lemma 3. □

Recall now that L_{SS} does nothing more than invert the marked/ unmarked dichotomies of L_A – this being one of the stated reasons for investigating it given in Humberstone (1982). Moreover, therein it is shown that L_A and L_{SS} are expressively equivalent, and those expressive equivalence results, much like the results we give above, show that the two languages are expressively equivalent when truth in a model is thought of as d-truth. Given, then, that expressive equivalence is a transitive relation between languages, it follows that L_S and L_{SS} are also expressively equivalent. But from this it seems to follow that, Wehmeier's own protestations to the contrary notwithstanding, the only difference between L_A and L_S is over which mood is marked and which is unmarked – making them notational variants in the manner alluded to by Wehmeier. That is, the only difference between the two languages becomes whether we are marking the indicative or subjunctive moods, and the devices which are using to do so.

4 Subjunctively Closed Formulas

One possible objection one might make at this point is that, while the translation τ_1 yields subjunctively closed formulas, its auxiliary translation τ_S does not. Consequently, when we use τ_S to show that L_A and L_S are expressively equivalent we are doing so in terms of formulas which are not subjunctively closed. For example, the τ_S-translation of the formula p, will be p^s, in which we have a subjunctive variable not in the scope of a modal operator. As formulas which are not subjunctively closed are second class citizens in Wehmeier's

Wehmeier (2004)) only occur when we think of truth in a model as real-world truth – otherwise they are no different from the cases where (in L_S) we can have p and p^s have the same truth value when our subjunctive world parameter becomes the actual world.

subjunctive modal language, in the same way that open formulas are second class citizens in predicate logic, one might object that this translation does not show that the two languages are properly expressively equivalent, as it does not preserve the implicit restriction that the formula ψ in (3) above should be subjunctively closed.

As it turns out, though, this requirement is in tension with taking truth in a model for formulas of L_A as truth at the designated world. That is, if we are to take truth in a model for formulas of L_A as d-truth, it turns out that there can be no translation τ which will map formulas of L_A to subjunctively closed formulas in L_S such that for all formulas $\varphi \in L_A$ we have φ and $\tau(\varphi)$ true in the same models – from which it follows that L_A and the subjunctively closed fragment of L_S – henceforth L_S^* – are not expressively equivalent. To show this consider the following two d-models.

- $W := \{x, y\}$.
- $V(p) := \{x\}$.
- $\mathcal{M} = \langle W, x, x, V\rangle$.
- $\mathcal{M}' = \langle W, y, x, V\rangle$.

By Lemma 1 it follows that the same subjunctively closed formulas are true throughout both $\mathcal{M}$ and $\mathcal{M}'$. As a result we have that for all subjunctively closed formulas ψ that $\mathcal{M} \Vdash^r \psi \iff \mathcal{M}' \Vdash^r \psi$, and also that $\mathcal{M} \Vdash^d \psi \iff \mathcal{M}' \Vdash^d \psi$. It is also easy to see that $\mathcal{M} \models_x p$ (and hence $\mathcal{M} \models^d p$) while $\mathcal{M}' \models_y \neg p$ (and hence $\mathcal{M} \models^d \neg p$). This means that there can be no translation τ such that p and $\tau(p)$ are d-true in the same models, which gives us the following result.

Theorem 2. *Let truth in a d-model for formulas of L_A be d-truth, and truth in a d-model for formulas of L_S be either d-truth or real-world truth. Then L_A and the subjunctively closed fragment of L_S (L_S^*) are not expressively equivalent.*

Proof. We will focus on the case where truth in a model for formulas in the subjunctively closed fragment of L_S is d-truth. Recall that, in particular, in order for L_A and L_S^* to be expressively equivalent we would need to have the following.

$$\forall \varphi \in L_A \exists \psi \in L_S^* (\forall \mathcal{M} : \mathcal{M} \models^d \varphi \iff \mathcal{M} \Vdash^d \psi).$$

One instance of this will be where $\varphi = p$, which would require that there be some formula $\psi_p \in L_S^*$ such that for all models $\mathcal{M}$, we have that $\mathcal{M} \models^d p \iff \mathcal{M} \Vdash^d \psi_p$. But the models $\mathcal{M}$ and $\mathcal{M}'$ above agree on all subjunctively closed formulas (i.e. all formulas in L_S^*), but disagree over whether $p \in L_A$ is true in them. So there can be no such formula ψ_p, and hence the two languages cannot be expressively equivalent. $\square$

This suggests that, if we are being charitable concerning our notion of truth in a model, that the differences regarding the marked-unmarked dichotomies in the two languages, are there simply by fiat – put there by placing the focus (justifiable or not) on subjunctively closed formulas in L_S. If we are to think of truth in a model for formulas in the actuality language as d-truth then the proponents of the subjunctive language must make one of a number of choices. On the one hand they could just bite the bullet concerning Theorem 2, and admit that what they care about is the subjunctively closed fragment of L_S, living with the fact that their language is (formally speaking at least) expressively weaker than the actuality language. The task here would be, of course, to try and explain away this apparent expressive inadequacy in some way. On the other hand they could take the moral from Theorem 2 to heart, and admit that what they care about more is the expressive equivalence between their language and the actuality language, thus placing less emphasis on subjunctively closed formulas – possibly banishing them to the realm of the pragmatics of modal language.[8]

There is of course one final option for the proponent of the subjunctive language. Our discussion has largely assumed the desirability of the proponent of the actuality language thinking of truth in a model in terms of d-truth in a d-model (and derivatively, validity as general validity). That we should do so is defended by Hanson, but the proponent of the subjunctive language could deny this – claiming that the only sensible or appropriate notion of truth in a model and validity for formulas in the actuality language is real-world truth/validity. Given how much rides on such choices for validity and truth in a model for the actuality language such issues deserve a more thorough investigation.

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References

Crossley, J. N., & Humberstone, L. (1977). The Logic of "Actually". *Reports on Mathematical Logic*, **8**, 11–29.

Davies, M., & Humberstone, L. (1980). Two notions of necessity. *Philosophical Studies*, **38**, 1–30.

Hanson, W. H. (2006). Actuality, Necessity, and Logical Truth. *Philosophical Studies*, **130**, 437–459.

[8]The proponent of L_S could also argue that the proponent of L_A should also be concerning themselves solely with the analogue of the subjunctively closed fragment for that language – with every propositional variable either in the scope of an A or a $\Diamond$ – call such formulas A-*subjunctively closed*. Of course, one might think that the expressibility of formulas such as $\mathsf{A}p \leftrightarrow p$ are one of the appeals of this choice of language (this sentence being a nice example of something which is a priori true), a consideration which lies behind the example discussed in fn.6 . That said, the translations above will still map A-subjunctively closed formulas of L_A to subjunctively closed formulas of L_S – allowing us to recapture the expressive equivalence of the two languages while maintaining that truth in a model for L_A is d-truth.

Hazen. A. (1978). The eliminability of the actuality operator in propositional modal logic. *Notre Dame Journal of Formal Logic*, **19**, 617–622.

Humberstone, L. (1982). Scope and Subjunctivity. *Philosophia*, **12**, 99–126.

Humberstone, L. (2004). Two-Dimensional Adventures. *Philosophical Studies*, **118**, 17–65.

Wehmeier, K. F. (2004), In the Mood, *Journal of Philosophical Logic*, **33**, 607–630.

Wehmeier, K. F. (2005). Modality, Mood, and Description. (In R. Kahle (Ed.), Intentionality: An Interdisciplinary Discussion (pp. 187–216). Wellesley, Mass.: A.K. Peters.)